Discussion of

Real-time Portfolio Choice Implications of Asset Pricing Models

by Francisco Barillas and Jay Shanken

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2019 FMA Consortium on Factor Investing

Why should we care?

A decision maker $\mathcal{D}$ wants to model asset returns y based on a set of risk factors $\boldsymbol{z}$. A canonical approach is

$$y_t = \boldsymbol{\beta}'\boldsymbol{z}_t + \epsilon_t, \qquad \epsilon_t \sim \pi(0, \nu_t)$$

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- Exploit $\neq$ adaptivity to breaks.

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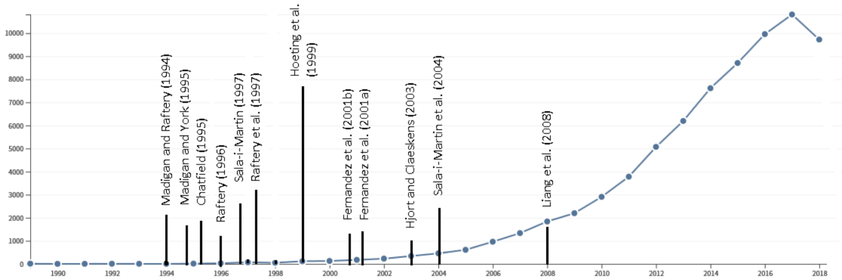

Total number of citations to papers with topic "model averaging" over years 1989-2018 in economics, finance, econometrics, statistics, etc. Google scholar returns over 52,500 papers in a search for "model averaging" and over 39,000 papers when searching for BMA.

This Paper

One of the problems in BMA is to figure out the "weight" ω_i, of each model $\mathcal{M}_i, i = 1, ..., N$, i.e.,

$$\omega_i \propto p\left(y_{t+1}|\mathcal{M}_i\right) p\left(\mathcal{M}_i\right)$$

Two ingredients:

- Marginal likelihood $p\left(y_{t+1}|\mathcal{M}_i\right)$.
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This paper:

Investigates the implications for factor-based portfolio allocations when ω_i's are used for model averaging.

My Comments:

Comment 1: BMA implementation.

Comment 2: Real-time portfolio choice implementation.

Comment 3: Optimal portfolios formation.

Comment 1: Bayesian Model Averaging

Remind: BMA is "optimal" only in an $\mathcal{M}$-closed setting, i.e., when the *true* model $\mathcal{M}^T$ is in the set of models considered, i.e., $\mathcal{M}^T \in \mathcal{M}$

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Suppose the true model is

$$y = x_1\beta_1 + x_2\beta_2 + \epsilon$$

and you estimate

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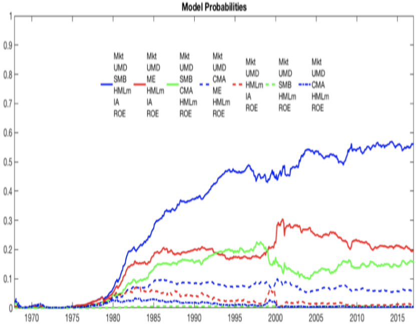
Comment:

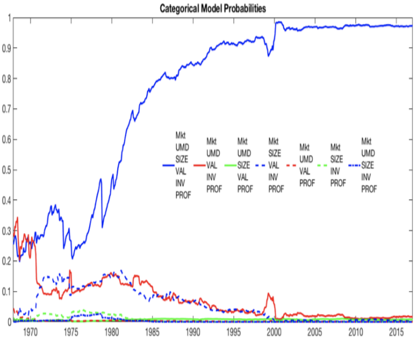
– Discuss the trade-off $\mathcal{M}$-closed vs. $\mathcal{M}$-open setting with BMA.

– In an $\mathcal{M}$-open setting model weights are the outcome of the decision problem (see Ch.9. Bernardo and Smith 1994), i.e., models can be selected based on the portfolio utility.

Comment 2: Real-Time Portfolio Allocation



(a) Model Probabilities



(b) Categorial Model Probabilities

In the paper:

- Expanding window; initial 20 years of data, from 01:1967 to 12:1986, then recursively add observations until the end of the sample.
- Allocations convergence towards the "best" model (in relative terms).

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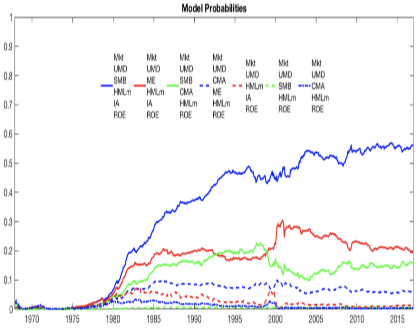
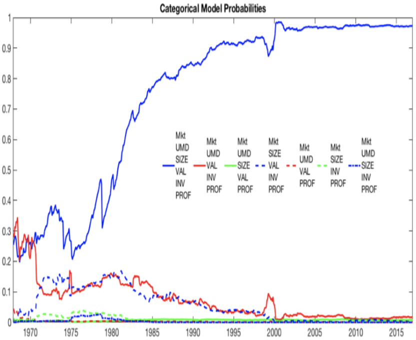


(c) Model Probabilities



(d) Categorial Model Probabilities

Comment:

- Rolling window; initial 20 years of data, from 01:1967 to 12:1986, then keep the same window until the end of the sample.
- Allocations won't convergence any more towards the "best" model.

Comment 3: Forming Optimal Portfolios

An investor maximizes wealth

$$\max_{\boldsymbol{w}} E_t \left[U \left(W_{t+1} \right) \right]$$

by choosing among N zero-cost risky portolios and a riskless asset, s.t.,

$$W_{t+1} = W_t \cdot \left[R_t^f + \boldsymbol{w}_t' f_{t+1} \right]$$

If $U \left(W_{t+1} \right)$ is quadratic only the first two moments matter

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If $U\left(W_{t+1}\right)$ is e.g., power utility, also the higher moments matter (see, e.g., Guidolin and Timmermann 2008).

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Comment:

– Perhaps look at how optimal portfolio investments change from higher moments within a power utility context.

Conclusion

Very interesting paper, recommended reading!!